Jet Propulsion

A simple guide to the aerodynamics and thermodynamic design and performance of jet engines

Third Edition

Nicholas CUMPSTY and Andrew HEYES

Reading notes by Aimery TAUVERON

Year 2016-2017
# Contents

1 Design of Engines for a New Efficient Aircraft  
1 The New Efficient Aircraft: Requirements and Background  
2 The Aerodynamics of the Aircraft  
3 The Creation of Thrust in a Jet Engine  
4 The Gas Turbine  
5 The Principle and Layout of Jet Engines  
6 Elementary Fluid Mechanics of Compressible Gases  
7 Selection of Fan Pressure Ratio, Specific Thrust and Bypass Ratio  
8 Dynamic scaling and dimensional analysis  
9 Turbomachinery: compressors and turbines  
10 Overview of the civil engine design  

II Engine Component Characteristics and Engine Matching  
11 Component characteristics  
12 Engine matching off-design  

III Appendix  
A Axial Turbine and Compressor Design  
B Stress Analysis, Material Design issues and Failure Analysis
Part I

Design of Engines for a New Efficient Aircraft
Chapter 1

The New Efficient Aircraft: Requirements and Background

Some background

Environmental issues

Commercial aspects of new large aircraft

Specification of the New Efficient Aircraft

<table>
<thead>
<tr>
<th>Normal max. passengers</th>
<th>280</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range at max. payload, R1 (nm)</td>
<td>3,000</td>
</tr>
<tr>
<td>Max. payload (tonne)</td>
<td>40.2</td>
</tr>
<tr>
<td>Max. take-off weight (tonne)</td>
<td>175</td>
</tr>
<tr>
<td>Empty weight (tonne)</td>
<td>100</td>
</tr>
<tr>
<td>Fuel burn at R1</td>
<td>27.8</td>
</tr>
<tr>
<td>Fuel burn per nm at R1</td>
<td>9.2</td>
</tr>
<tr>
<td>Cruise Mach no.</td>
<td>0.78</td>
</tr>
<tr>
<td>Cruise L/D</td>
<td>21.6</td>
</tr>
<tr>
<td>Engine sfc (kg h⁻¹ kg⁻¹)</td>
<td>0.50</td>
</tr>
<tr>
<td>Wing span (m)</td>
<td>60</td>
</tr>
<tr>
<td>Wing area (m²)</td>
<td>304</td>
</tr>
<tr>
<td>Wing sweep</td>
<td>25</td>
</tr>
</tbody>
</table>

The units used

\[
1 \text{ lb} = 0.4536 \text{ kg} \\
1 \text{ ft} = 0.3048 \text{ m} \\
1 \text{ nm} = 1.852 \text{ km}
\]

- Speed of sound: \( a = \sqrt{\gamma RT} \), where \( T \) is the static temperature, \( \gamma \) is the ratio of the specific heats (1.40 for air), and \( R \) is the gas constant (0.287 kJ kg⁻¹ K⁻¹ for air).

The standard atmosphere

- Standard sea-level atmospheric conditions (ISA): \( T_{sl} = 288.15 \text{ K} \), \( p_{sl} = 1013 \text{ hPa} \), \( \rho_{sl} = 1.225 \text{ kg/m}^3 \).
- Linear temperature decrease (6.5 K per 1000 m) up to the tropopause (11,000 m), then constant (216.65 K).
Chapter 2

The Aerodynamics of the Aircraft

Payload versus range

Sizing the wing

- Lift coefficient $C_L = \frac{L}{\frac{1}{2} \rho A V^2}$; drag coefficient $C_D = \frac{D}{\frac{1}{2} \rho A V^2}$.

- $C_D = C_{D0} + \frac{K \nu}{\pi A R} C_L^2$. $C_{D0}$ is the consequence of viscous drag and separation effects.

Lift, drag, fuel consumption and range

- For steady level flight at small incidence: lift = weight and drag = thrust of the engines.
- For maximum range: optimise $ML/D$. For maximum endurance: optimise $L/D$.

Specific fuel consumption

- $sfc = \frac{\dot{m}_f}{\text{thrust}}$ kg s$^{-1}$ N$^{-1}$. $sfc$ is often expressed in kg hr$^{-1}$ kgf$^{-1}$.

Breguet range equation

- Range factor $H = \frac{m \text{LCV} (L/D)}{g}$.
- For a constant $H$: $m_{end}/m_{start} = \exp(-R/H)$ (with $m_{start}$ and $m_{end}$ the total aircraft mass at start and end of cruise respectively) i.e $R = -H\ln\left(\frac{m_{end}}{m_{start}}\right)$ (Breguet’s range formula).
- From the Breguet equation: $\frac{m_{fb}}{m_{pl}} = \frac{1}{R} \left(1 + \frac{m_{fb}}{m_{pl}}\right) (\exp(R/H) - 1)$ (with $m_{pl}$ the payload and $m_{fb}$ the mass of burnt fuel).

Selecting engine thrust for climb

- The critical condition for sizing modern engines is top-of-climb.
- If the aircraft is climbing at an angle to the horizontal $\theta$ ($\theta \sim 1^\circ$): $F_N/w = D/w + \sin \theta = 1/(L/D) + \sin \theta$.

Appendix: Specific Air Range and ICAO

- Specific Air Range $SAR = V/\dot{m}_f = H/M$. 
Chapter 3

The Creation of Thrust in a Jet Engine

Momentum change

- Net thrust $F_N = (\dot{m}_{air} + \dot{m}_f)V_j - \dot{m}_{air}V$.
- Gross thrust (i.e., thrust produced with the aircraft stationary): $F_G = (\dot{m}_{air} + \dot{m}_f)V_j$.
- The difference between gross and net thrust is called *ram drag*: $F_N = F_G - \dot{m}_{air}V$.

Propulsive efficiency

- Propulsive efficiency $\eta_p = \frac{2V}{V + V_j} \left( = \frac{\dot{m}_{air}V[V_j - V]}{(1/2) \dot{m}_{air} [V_j^2 - V^2]} \right)$.

Overall efficiency

- Thermal efficiency $\eta_{th} = \frac{\dot{m}_{air} [V_j^2 - V^2]/2}{\dot{m}_f \text{ LCV}}$.
- Overall efficiency $\eta_0 = \eta_p \cdot \eta_{th} = \frac{1}{\text{sscf}} \cdot \frac{V}{\text{LCV}}$. 
Chapter 4

The Gas Turbine

Gas turbine principles

• For the whole process: \( \dot{Q}_{\text{net}} - \dot{W}_{\text{net}} = \dot{m}_{\text{air}} c_p (T_5 - T_2) \).
• For the combustion process: \( \dot{m}_f LCV = \dot{m}_{\text{air}} c_p (T_4 - T_3) \).
• Data: \( LCV = 43 \text{ MJ kg}^{-1}, c_p = 1,005 \text{ J kg}^{-1} \text{ K}^{-1}, \gamma = 1.40, R = 287 \text{ J kg}^{-1} \text{ K}^{-1} \).

Isentropic efficiency and the exchange of work

• \( \eta_{\text{comp}} = \frac{T_3 - T_2}{T_3 - T_2} \) and \( \eta_{\text{turb}} = \frac{T_5 - T_4}{T_5 - T_4} \). For a commercial aircraft: \( \eta_{\text{comp}} \approx \eta_{\text{turb}} \approx 90\% \).

• In a compressor: \( \dot{W}_{\text{c}} = \frac{\dot{m}_{\text{air}} c_p (T_{3,\text{is}} - T_2)}{\eta_{\text{comp}}} = \frac{\dot{m}_{\text{air}} c_p T_{3,\text{is}} (r^{(\gamma-1)/\gamma} - 1)}{\eta_{\text{comp}}} \).

• In a turbine: \( \dot{W}_{\text{t}} = \eta_{\text{turb}} \dot{m}_{\text{air}} c_p (T_4 - T_{5,\text{is}}) = \eta_{\text{turb}} \dot{m}_{\text{air}} c_p T_4 (1 - r^{-1}/\gamma) \).

• \( \dot{W}_{\text{net}} = \dot{W}_{\text{turb}} - \dot{W}_{\text{comp}} \).

The gas turbine thermal and cycle efficiency

• \( \eta_{\text{cycle}} = \frac{\dot{W}_{\text{net}}}{\dot{m}_{\text{air}} c_p (T_4 - T_3)} \).

• \( \eta_{\text{th}} = \eta_{\text{cycle}} \).

The effect of working gas properties

• For a semi-perfect gas: \( \gamma \) and \( R \) are functions of temperature and composition (but not of pressure).

• For the gases occurring in a gas turbine, \( R \) may be assumed independent of both temperature and composition.

The gas turbine and the jet engine

APPENDIX: A brief summary of thermodynamics from an engineering perspective

• For an ideal gas of fixed composition, the enthalpy is a function of temperature only: \( dh = c_p dT \).

• For a reversible and adiabatic change in pressure: \( p/T^{\gamma/2} = \text{constant} \).
Chapter 5

The Principle and Layout of Jet Engines

The turbojet and the turbofan

- Used in early jets.
- Similar arrangements in many military jet engines (bypass ratio typically between 0.3 and 1.5).

The high bypass ratio engine

- Used in modern airliners.
- Bypass ratio of 5 or more, now typically more than 9.
- Two-shaft with booster stages (Pratt & Whitney, General Electric), three-shaft (Rolls-Royce).

Turbine inlet temperature

- Increasing $T_4$ allows for better thermal efficiency, and more power available to the LP turbine.
- $T_4$ is limited by material (turbine blades), hence the use of cooling (using 15 to 25% of the core air flow).
- Cooling effectiveness: $\epsilon = \frac{T_g - T_m}{T_g - T_c}$, where $T_g$ is the temperature of the hot gas stream, $T_m$ the temperature of the metal, $T_c$ the temperature of the cooling air. Current state-of-the-art: $0.6 \leq \epsilon \leq 0.7$. 

Chapter 6

Elementary Fluid Mechanics of Compressible Gases

Incompressible and compressible flow

- Bernoulli’s equation (incompressible flows): \((1/2) V^2 + p/\rho = p_0/\rho = \text{constant along a streamline.}\)

Static and stagnation conditions

- \(c_p T + V^2/2 = c_p T_0\) with \(T_0\) the stagnation temperature.
- \(T_0/T = 1 + \frac{\gamma-1}{2} M^2\).
- \(p_0/p = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/(\gamma-1)}\) since the acceleration/deceleration between the static and stagnation states is assumed isentropic.

The nozzle

- For adiabatic flow: \(V = \sqrt{2 c_p (T_0 - T)}\), where \(T\) is the local temperature of the air and \(T_0\) the stagnation temperature (uniform through the flow). Since the flow can be assumed loss-free: \(V = \sqrt{2 c_p \left(1 - (p_a/p_0)^{(\gamma-1)/\gamma}\right)}\), where \(p_a\) is the exit static temperature.

The choked nozzle

- The mass flow rate per unit area is \(\dot{m}/A = \rho V\). It is maximum for \(M = 1\).

Normalised mass flow per unit area

- The non-dimensional mass flow rate per unit area is \(\bar{m} = \frac{\dot{m} \sqrt{c_p T_0}}{A p_0}\).
- \(\bar{m}\) is a function of the Mach number \(M\) and the specific heat ratio \(\gamma\) only. \(\bar{m} = M \frac{2}{\sqrt{\gamma-1}} \left(1 + \frac{\gamma-1}{2} M^2\right)^{-(\gamma+1)/(2(\gamma-1))}\).
- If the Mach number \(M\) is the result of an adiabatic and reversible expansion from stagnation pressure \(p_0\) to static pressure \(p\) (e.g. in a nozzle): \(\bar{m}(p/p_0, \gamma) = \frac{2}{\sqrt{\gamma-1}} \sqrt{2 \left((p/p_0)^{2/\gamma} - (p/p_0)^{(\gamma+1)/\gamma}\right)}\).

The fan, the bypass stream and the propulsive nozzle

- For modern high bypass engines, the bypass stream accounts for about 90% of the thrust.
Chapter 7

Selection of Fan Pressure Ratio, Specific Thrust and Bypass Ratio

Introduction

- For a pure turbojet: \( \eta_0 = \eta_p \eta_{th} \).
- For a bypass engine: \( \eta_0 = \eta_p \eta_{th} \eta_{tr} \), where \( \eta_{tr} \) represents the transfer efficiency (extracting power in the LP turbine to increase the pressure in the bypass flow).

Fan pressure ratio and bypass ratio

- The bypass ratio is a common descriptor of the engine type.
- Fan pressure ratio is a good descriptor of engine. An alternative is to use the specific thrust: \( \text{specific thrust} = F_N/\dot{m}_{air} = V_j - V \).

Engine layout and station numbering

Engine overall specification

The specification of fan pressure ratio

- In the bypass nozzle: \( \frac{p_{013}}{p_a} = f_{pr} \frac{p_{02}}{p_{a}} \), \( T_{013} = T_{02} \left( 1 + \frac{\eta_{tr}(\gamma-1)/\gamma - 1}{\eta_f} \right) \).
- The bypass jet velocity is: \( V_{jb} = \sqrt{2 c_p (T_{013} - T_9)} = \sqrt{2 c_p T_{013} \left( 1 - (p_a/p_{013})^{(\gamma-1)/\gamma} \right)} \).
- The core jet velocity is: \( V_{jc} = \sqrt{2 c_p T_{05} \left( 1 - (p_9/p_{05})^{(\gamma-1)/\gamma} \right)} \).
- The core and bypass jet velocity are of the same order of magnitude, and may thus be assumed equal: \( V_{jb} = V_{jc} \).
- The power of the LP turbine is equal to the power taken by the fan and booster: \( \dot{m}_c c_p (T_{045} - T_{05}) = \dot{m}_c c_p (T_{023} - T_{02}) + bpr \dot{m}_c c_p (T_{013} - T_{02}) \).

Procedure for calculation:

1. Choose the fan pressure ratio, which determines bypass jet velocity.
2. Choose the ratio of core jet velocity to bypass jet velocity.
3. Guess a value for LP turbine pressure ratio, \( p_{045}/p_{05} \).
4. From \( p_{045}/p_{05} \) find \( T_{05} \) and \( p_{05}/p_a \).
5. Compute core jet velocity \( V_{jc} \) and compare it to bypass jet velocity \( V_{jb} \). If \( V_{jc} \) is too large increase \( p_{045}/p_{05} \) and return to step (4).
6. From $p_{045}/p_{05}$ and $T_{045}$ find LP shaft power and then compute bypass ratio.
7. Compute gross thrust and net thrust per unit mass flow through the core.

The impact of fan pressure ratio

- A decrease in fan pressure ratio causes an increase in bypass ratio, and in LP turbine pressure ratio.
- Reducing fan pressure ratio causes a strong increase of gross thrust. Net thrust varies much less.
- Specific thrust increases with fan pressure ratio.

The bare engine and the effect of the nacelle

- $D_{nac} = k V (F_N/X)$, where $X$ is the engine specific thrust, and $k$ an empirical constant ($k = 0.04$ in general).
- $F_{N, effective} = F_{N, bare} (1 - k V/X)$. For the NEA: $F_{N, effective} = F_{N, bare} (1 - 9.25/X)$.

Effect of engine weight and selection of fan pressure ratio

- $W_{engine} \propto d^{2.4}$. For a modern engine with fan diameter of 3 m: $W_{engine} \approx 12$ t.
- The thrust after deducing the drag attributable to engine weight is: $F_{N, corrected} = F_{N, effective} - W_{engine}/(L/D)$.

The effect of unequal bypass and core jet velocity
Chapter 8

Dynamic scaling and dimensional analysis

Engine variables and dependence

- The primary control to the engine is the variation of $\dot{m}_f$.
- Any variable in the engine can be expressed as a function of $\dot{m}_f$, $p_a$, $T_a$ and $V$ (flight speed).
- For modern high-bypass engines, the bypass nozzle is choked only at cruise while the core nozzle is usually never choked.

Non-dimensional variables of the engine

- For mass flow: $\bar{m} = \frac{\dot{m}_{\text{air}} \sqrt{c_p T_0}}{D^2 p_0}$.

Non-dimensional treatment of thrust for choked nozzle

- For thrust: $\frac{F_G + p_a A_N}{D^2 p_0}$.

Practical scaling parameters

- For mass flow: $\frac{\dot{m}_{\text{air}} \sqrt{T_0}}{p_0}$.
- For fuel flow: $\frac{\dot{m}_f}{\sqrt{T_0^2 p_0}}$.
- Non-dimensional speed: $\frac{N}{\sqrt{\theta}}$ where $\theta = \frac{T_0}{T_{02,\text{ref}}}$ with $T_{02,\text{ref}}$ a reference temperature (typically 288 K).
- Corrected mass flow: $\frac{\dot{m}_{\text{air}} \sqrt{T_0}}{\delta}$ where $\theta = \frac{T_0}{T_{02,\text{ref}}}$ and $\delta = \frac{p_0}{p_{02,\text{ref}}}$.
- Specific fuel consumption: $sfc = \frac{\dot{m}_f}{F_N}$. 

<table>
<thead>
<tr>
<th>Non-dimensional mass flow rate $\bar{m}$</th>
<th>$\frac{\dot{m}_{\text{air}} \sqrt{c_p T_0}}{D^2 p_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only required if the propulsive nozzle is unchoked</td>
<td>$\frac{\dot{m}_f}{\sqrt{T_0^2 p_0}}$</td>
</tr>
<tr>
<td>Non-dimensional shaft speed $N$ $\sqrt{\theta}$</td>
<td>$\frac{\dot{m}_f LCV}{\sqrt{c_p T_0^2 D^2 p_0}}$</td>
</tr>
<tr>
<td>Non-dimensional fuel flow rate $\dot{m}_f$</td>
<td>$\frac{\dot{m}<em>{\text{air}} V</em>{19} + p_{19} A_N}{D^2 p_0}$</td>
</tr>
<tr>
<td>Dimensionless gross thrust</td>
<td>$\frac{F_G + p_a A_N}{D^2 p_0}$</td>
</tr>
</tbody>
</table>
Loss of thrust from an engine

- An aircraft must be able to take-off, climb and then land safely even if an engine fails during take-off (just before rotation), without the pilot altering the throttles.

- Loss of thrust during cruise: the altitude must decrease to allow for an increase in thrust. This entails a decrease in speed and in range.
Chapter 9

Turbomachinery: compressors and turbines

The blades for axial compressors and turbines

- Let’s define: $\alpha_1$ gas angle into blade, $\alpha_2$ gas angle out of blade, $\beta_1$ blade metal angle in, $\beta_2$ blade metal angle out.
- Incidence $i = \alpha_1 - \beta_1$, deviation $\delta = \alpha_2 - \beta_2$. Generally, $i = 0$ by design. For modern compressor blades $\delta = 0$.
- Velocity $V_1 = V_2 / \cos \alpha_1$ at inlet, $V_2 = V_2 / \cos \alpha_2$ at outlet.
- For a compressor: $|\alpha_2| < |\alpha_1|$ thus $V_2 < V_1$, the flow is decelerated.
- For a turbine: $|\alpha_2| > |\alpha_1|$ thus $V_2 > V_1$, the flow is accelerated.

Frames of reference

- For the rotor: moving or relative frame of reference, the velocities are described as relative.
- For the stator: stationary frame of reference, the velocities are described as absolute.

The Euler work equation

- The torque is $T = \dot{m} \left( r_2 V_{\theta_2} - r_1 V_{\theta_1} \right)$. Thus the power in the rotor is $\dot{W} = T \Omega = \dot{m} \Omega \left( r_2 V_{\theta_2} - r_1 V_{\theta_1} \right) = \dot{m} \left( U_2 V_{\theta_2} - U_1 V_{\theta_1} \right)$ with $\Omega$ the rotation speed of the rotor, $U_1$ and $U_2$ the speed of the blade row at inlet and outlet.
- $\dot{W} = \dot{m} \Delta h_0$ thus $\Delta h_0 = U_2 V_{\theta_2} - U_1 V_{\theta_1}$. If $r_1 = r_2$ then $\Delta h_0 = U \left( V_{\theta_2} - V_{\theta_1} \right)$.
- Work coefficient $\psi = \frac{\Delta h_0}{\dot{m} \dot{V}} = \frac{\Delta V}{\dot{m} \dot{V}}$.

Flow coefficient and work coefficient

- Flow coefficient $\phi = \frac{V_x}{U}$.
- For compressors $\phi \approx .4 - .75$, for core turbines: $\phi \approx .5 - .65$, for LP turbines $\phi \approx .7 - 1.1$.
- The Smith chart gives an experimental relation between the work and flow coefficients for turbines.
- There is no such diagram for compressors. Generally $\psi \approx .35 - .5$.
- The fan is a special compressor stage. The axial flow has a relatively high Mach number (about .6), the mass flow per unit area is about 85% of that required to choke the annulus were there no fan blades between hub and casing. The maximum relative tip Mach number for the fan is 1.6 for a pressure ratio of 1.8, 1.3 for a pressure ratio of 1.5.
Figure 9.1: Axial compressor and turbine stages, showing the velocity triangles (images from *Jet Propulsion*, p. 124 and 129)

The axial turbine

- Euler work equation: \[ \Delta h^0 = U (V_{\theta 2} - V_{\theta 3}) = U (V_{\theta 2}' - V_{\theta 3}'). \] If the axial velocity is constant: \[ \Delta h^0 = U V_x (\tan \alpha_2 - \tan \alpha_3) = U V_x (\tan \alpha_2' - \tan \alpha_3'). \]

- Zweifel loading coefficient \[ Z_w = \frac{F_t}{c_x (p_{0,in} - p_{out})}, \] which becomes \[ Z_w = \frac{2 s}{c_x \cos^2 \alpha_{out} (\tan \alpha_{out} - \tan \alpha_{in})}. \]

<table>
<thead>
<tr>
<th>Aspect ratio h/c</th>
<th>.75-2.5</th>
<th>for LP turbines up to about 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch-chord ratio s/c</td>
<td>.75-1.5</td>
<td>for LP turbines up to 1.0</td>
</tr>
<tr>
<td>Zweifel coefficient</td>
<td>&lt;.8</td>
<td>for LP turbines .7-1.1</td>
</tr>
<tr>
<td>Flow coefficient ( \phi = V_x / U )</td>
<td>.5-.65</td>
<td>for LP turbines up to 2.4</td>
</tr>
<tr>
<td>Work coefficient ( \psi = \Delta h_0 / U^2 )</td>
<td>.8-2.0</td>
<td>for LP turbines down to .7</td>
</tr>
</tbody>
</table>

The axial core compressor

- Euler work equation: \[ \Delta h^0 = U (V_{\theta 2} - V_{\theta 1}) = U (V_{\theta 2}' - V_{\theta 1}'). \] If the axial velocity is constant: \[ \Delta h^0 = U V_x (\tan \alpha_2 - \tan \alpha_1) = U V_x (\tan \alpha_2' - \tan \alpha_1'). \]

- Diffusion coefficient \( DF = (1 - V_2/V_1) + \Delta V_\theta / (2 \sigma V_1) \). If the flow is at constant radius and constant axial velocity \( DF = (1 - \cos \alpha_1 / \cos \alpha_2) + (\tan \alpha_2 - \tan \alpha_1) \cos \alpha_1 / (2 \sigma) \).

<table>
<thead>
<tr>
<th>Aspect ratio h/c</th>
<th>.75-2.5</th>
<th>for fans up to about 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch-chord ratio s/c</td>
<td>.6-1.5</td>
<td>for LP compressors (booster)</td>
</tr>
<tr>
<td>Diffusion factor DF</td>
<td>&lt;.5</td>
<td>perhaps higher in LP compressor (booster)</td>
</tr>
<tr>
<td>Flow coefficient ( \phi = V_x / U )</td>
<td>.45-.75</td>
<td>perhaps higher in LP compressor (booster)</td>
</tr>
<tr>
<td>Work coefficient ( \psi = \Delta h_0 / U^2 )</td>
<td>.35-.5</td>
<td>perhaps higher in LP compressor (booster)</td>
</tr>
<tr>
<td>Blade or vane exit Mach number</td>
<td>&lt;1.0</td>
<td>( M_{rel} &gt; 1 ) at tips of fans and front stages</td>
</tr>
</tbody>
</table>

Spanwise effects

- For a free-vortex blade: \( r V_\theta = \text{constant} \).
Chapter 10

Overview of the civil engine design
Part II

Engine Component Characteristics and Engine Matching
Chapter 11

Component characteristics

Gas properties in the aircraft gas turbine

- In previous chapters, the specific heat capacity $c_p$ and specific heat ratio $\gamma$ were assumed equal to that of air, and constant with temperature and pressure.
- For a gas turbine combustor at maximum thrust, the equivalence ratio is $\phi \approx 0.3$ for a civil engine, whereas it is about 1 for a military engine (with afterburner on).

The propulsive nozzle

- For a choked nozzle: 
  $$\dot{m}_{\text{choke}} = \left( \frac{\dot{m} \sqrt{c_p T_0}}{A_0} \right)_{M=1.0} = \frac{\gamma}{\sqrt{\gamma - 1}} \left( \frac{\gamma + 1}{2} \right)^{\frac{\gamma + 1}{\gamma - 1}}.$$
- For pure air (e.g. in the bypass nozzle): $\gamma \approx 1.4$ thus $\dot{m}_{\text{choke}} \approx 1.281$.
- For combustion products (in a turbine or in the core nozzle): $\gamma \approx 1.3$ thus $\dot{m}_{\text{choke}} \approx 1.389$.

The fan

- A specialised compressor, with pressure ratios up to 1.8 (up to 4 with two or three stages in military engines).
- Corrected mass flow $\dot{m}_{\text{corr}} = \dot{m} \sqrt{\frac{T_0}{\delta}}$, corrected rotational speed $N_{\text{corr}} = \frac{N}{\sqrt{\delta}}$ with $\delta = p_{02}/p_{02\text{ref}}$ and $\theta = T_{02}/T_{02\text{ref}}$ (typically $p_{02\text{ref}} = 1.01$ bar and $T_{02\text{ref}} = 188$ K).
- Stall or surge line: maximum pressure rise a compressor can produce at a given rotational speed.

The core compressor

- Polytropic efficiency $\eta_p$: $\delta h_0 = c_p \delta T_0 = \delta p_0 / \eta_p p_0 = R T_0 / \eta_p p_0 \delta p_0$. For a compressor $\frac{T_{02}}{T_{01}} = \left( \frac{p_{02}}{p_{01}} \right)^{\frac{\gamma - 1}{\gamma}}$. For a turbine $\frac{T_{02}}{T_{01}} = \left( \frac{p_{02}}{p_{01}} \right)^{\frac{\gamma - 1}{\gamma \eta_p}}$.
- Non-dimensional mass flow based on outlet conditions: $\dot{m}_3 = \dot{m}_{23} \left( \frac{p_{03}}{p_{01}} \right)^L \frac{A_{03}}{A_{01}}$ where $L = 1 - \frac{\gamma - 1}{\gamma \eta_p}$.
- Off-design operation of multi-stage compressors.

The combustor

The turbine

- Turbines can often be considered to behave like a choked nozzle.
- Turbine cooling: up to 20% of the air entering the core compressor.
Chapter 12

Engine matching off-design

Assumptions and simplifications

• Several constraints have to be satisfied:

  1. The rotational speed of the compressor and turbine must be equal on each shaft.

  2. The mass flow through the compressor and turbine must be equal (bleeds and mass flow of fuel are neglected here).

  3. The power output of the turbine and the power input of the compressor must be equal on each shaft (losses and power take-offs are neglected here).

  4. The pressure rise in the compression processes and the pressure drop in the expansion processes must be equal.

• The non-dimensional mass flow is assumed equal in all turbines. Their efficiencies are assumed constant and independent of rotational speed.

A single-shaft turbojet engine

A two-shaft turbojet engine

The high-bypass turbofan engine

Application to a two-shaft high bypass turbofan

A three-shaft high bypass turbofan engine
Part III

Appendix
Appendix A

Axial Turbine and Compressor Design

This chapter is based on lecture notes by Ricardo Martinez-Botas.

Axial turbine design

- Work production: \( \dot{w}/\dot{m} = U V_x (\tan \alpha_2 - \tan \alpha_3) \).
- The turning \( \alpha_2^{rel} - \alpha_3^{rel} \) is typically 120°.
- Stage loading coefficient \( \psi = \frac{\Delta h_0}{\Delta h_{rotor}} \). Flow coefficient \( \phi = \frac{V_x}{U} \).
- Typically: \( 0.5 < \phi < 0.65 \) and \( 0.9 < \psi < 1.0 \).
- Smith chart: plot of iso-efficiency lines in the \( \psi - \phi \) plane (experimental data).
- Reaction \( R = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = \frac{\Delta h_{rotor}}{\Delta h_{0,stage}} \). Most commonly: \( R = 0.5 \). If \( R = 0 \): reaction turbine. If \( R = 1 \): impulse turbine.
- Pitch-to-chord ratio \( s/c \): typically between 0.5 and 1.
- Zweifel’s criterion: the best compromise is achieved for a tangential lift coefficient of 0.8, that is \( 0.8 = C_l = 2 (s/c) \cos^2 \alpha_2 (\tan \alpha_1 - \tan \alpha_2) \).
- Blade dimensions: typically \( h/c \) is between 2 and 3 for the rotor, between 1 and 2 for the stator. The blade thickness would be 10-20% of the chord.

Axial compressor design

- Work production: \( \dot{w}/\dot{m} = U V_x (\tan \alpha_1^{rel} - \tan \alpha_2^{rel}) \).
- The turning \( \alpha_1^{rel} - \alpha_3^{rel} \) is typically less than 45° (to avoid flow separation).
- Stage loading coefficient \( \psi = \frac{\Delta h_0}{\Delta h_{stage}} \). Flow coefficient \( \phi = \frac{V_x}{U} \).
- Typically: \( 0.4 < \phi < 0.7 \) and \( 0.35 < \psi < 0.5 \).
- Reaction \( R = \frac{\Delta h_{stage}}{\Delta h_{rotor}} = \frac{\Delta h_{stage}}{\Delta h_{0,stage}} \). Most commonly: \( R = 0.5 \).
- De Haller criterion: \( V_2^{rel}/V_1^{rel} < 0.72 \) and \( V_3/V_2 < 0.72 \).
- Lieblen diffusion factor: \( DF = \left(1 - \frac{V_2}{V_1}\right) + \frac{1}{2} \frac{V_{\theta,2} - V_{\theta,1}}{V_1} (s/c) \). Typically \( DF < 0.45 \).
Derivation of the radial equilibrium equation

- Radial static pressure gradient: \( \frac{\partial p}{\partial r} = \frac{1}{r} \rho V^2_\theta \), assuming \( V_r = 0 \) and that the curvature of the streamline in the \( x - r \) plane is negligible.

- Total enthalpy: \( h_0 = h + \frac{1}{2} (V^2_x + V^2_\theta) \).

- Since \( T \, ds = dh - \frac{dp}{\rho} \cdot \frac{dh}{dr} = T \, ds + V_x \, \frac{dV_x}{dr} + V_\theta \, \frac{d(rV_\theta)}{dr} \).

- Assuming radially uniform loss (\( \frac{ds}{dr} = 0 \)), radially uniform work input (\( \frac{dh}{dr} = 0 \)) and uniform axial velocity, we get: \( rV_\theta = \text{constant} \). This is known as the vortex equation.

- Moreover: \( \Delta h_0 = U (V_{\theta,2} - V_{\theta,1}) = \Omega \, r \, (V_{\theta,2} - V_{\theta,1}) \).

Reaction

- Reaction \( R = \frac{\Delta h_{\text{rotor}}}{\Delta h_{\text{stage}}} = \frac{\Delta h_{\text{rotor}}}{\Delta h_{0,\text{stage}}} \).

- For a turbine stage: \( R = \frac{C_2}{\pi U} (\tan \beta_2 + \tan \beta_3) \).

- For a compressor stage: \( R = \frac{C_1}{\pi U} (\tan \beta_1 + \tan \beta_2) \).
Appendix B

Stress Analysis, Material Design issues and Failure Analysis

This chapter is based on lecture notes by Dr Catrin M. Davies.

Stress Analysis of Aircraft Engine Components

Fan and Compressor Components

- Fan discs: react to centrifugal loads from the fan blades, provide attachment for the LP shaft, withstand impact loads.
- Compressor discs: subjected to high thermal stresses due to the temperature gradient between its bore and rim.
- Casings: must maintain acceptable levels of tip clearance, and resist in the event of fan-blade-off.

Combustor

Turbine Components

- Turbine discs: critical parts, manufactured from nickel alloy forgings.
- Turbine blades: operation at high temperature (manufactured from nickel alloy, use of thermal barrier coating), high centrifugal loads due to rotation, bending stresses from the gas flow, must withstand fatigue, thermal shock, corrosion and oxidation, creep.

Shafts Transmit torque and axial loads from the turbine to the compressor, must bear bending stresses. Usually made of high strength steels (treated to avoid corrosion).

Material Issues and Failure Modes

Yielding, fatigue, fracture, creep, corrosion, erosion, oxidation.

Yielding (Plastic Deformation) To avoid permanent deformations: \( \sigma_{eq} < \sigma_Y \) with:

- \( \sigma_{eq} = \frac{1}{\sqrt{2}} \left( (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right)^{1/2} \) (von Mises yield condition), or
- \( \sigma_{eq} = \frac{1}{2} \max(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|) \) (Tresca yield condition).

For single crystal and directionally solidified materials, the yielding behaviour is anisotropic and these yielding criterion do not apply.
Fatigue

- Fatigue failure mechanisms: crack initiation (usually at a notch or surface discontinuity), crack growth, ultimate rupture.
- High cycle fatigue may be caused by aerodynamic excitation, mechanical vibration and airfoil flutter. There are three basic factors for fatigue failure: a sufficiently high maximum tensile stress $\sigma_{\text{max}}$, a sufficiently large variation in stress levels $\sigma_a$ and a sufficiently large number of stress cycles $N$.
- The S-N curve in the high cycle region is described by the Basquin equation: $N_f = \frac{C}{\sigma_p^2}$ with $C$ and $p$ experimental constants.
- Low cycle fatigue (during each major cycle). It is described by the Coffin-Manson law: $\Delta \epsilon_p^2 = \epsilon_f' (2N)^c$ with $\epsilon_f'$ the fatigue ductility coefficient, $N$ the number of cycles to failure and $c$ a fatigue ductility exponent (for many materials $-0.7 < c < -0.5$).
- Thermal-mechanical fatigue. TMF occurs during the start-stop cycle of the engine.
- Key influencing parameters on fatigue: stress concentrating geometric features or cracks, residual stresses (tensile residual stresses are detrimental to fatigue as they increase the mean stress, compressive residual stresses reduce the net tensile stresses and may improve fatigue life), orthotropic material properties, surface finish, other damage (creep, oxidation, corrosion, material coatings, foreign object damage, erosion and wear...).
- Cumulative fatigue damage. Miner’s law predicts failure will occur if $D_f > 1$, defining $D_f = \sum \frac{N_i}{N_{fi}}$, with $N_i$ the number of cycles of type $i$ and $N_{fi}$ the number of cycles to failure in a cycle of type $i$.

Creep Deformation and Failure Mechanisms

- Creep occurs when metals are subjected to stresses at temperature greater than 30% of their absolute melting temperature.
- Two types of creep mechanisms in polycrystalline materials: dislocation creep and diffusion creep.
- Stages of creep: primary (creep strain rate decreases), secondary (constant creep strain rate), tertiary (accelerating creep strain rate).
- Power-law representations of creep. For steady-state (secondary) creep strain rate: $\dot{\epsilon}_s = C e^{-Q/RT} \sigma^n$ where $C$ is a constant, $Q$ the activation energy, $\sigma$ the equivalent stress and $n$ the power-law creep stress exponent. For constant temperature, this becomes: $\dot{\epsilon}_s = \dot{\epsilon}_0 \left( \frac{\sigma}{\sigma_0} \right)^n = A \sigma^n$.
- Creep rupture and rupture strength. The time to rupture can often be approximated by a power law: $t_r = B_r \sigma^{\nu_r}$ with $B_r$ and $\nu_r$ temperature-dependent constants.
- Creep rupture life prediction. The Larson-Miller parameter is: $P_{LM} = T (\log t_r + C)$ with $C$ a constant $\sim 20$ for most materials). It depends only on stress.
- Total deformation $\epsilon_{\text{total}} = \epsilon^e + \epsilon^p + \epsilon^c(t)$.
- Creep damage models: $D_c = \sum_{\sigma,T} \frac{\dot{\epsilon}^c(\sigma,T)}{\epsilon_f(\sigma,T)}$.

Combined Creep-Fatigue Interactive Failure Materials failure may be considered to occur when $D_T = 1$ with $D_T = D_f + D_c$.

Fracture Brittle fracture occurs when the maximum stress intensity factor in the component $K = Y \sigma \sqrt{a}$ reaches the material fracture toughness $K_C$.
Fatigue Crack Growth  Fatigue crack growth is assumed to follow Paris’ law: \[ \frac{da}{dN} = B \Delta K^m \] with \( \Delta K = K_{max} - K_{min} \), \( B \) and \( m \) material parameters.

Corrosion, Erosion and Oxidation

- Oxidation: reaction between the coating (or the base alloy) and the oxidants in the hot gas.
- Corrosion: results from the presence of salt contaminants that combine to form molten deposits.

Lifetime Design Considerations and Criterion

Failure Analysis of Components Example: Failure of a Turbine Disc of an Aero Engine

Materials Used in Aircraft Engine

- Fan: Ti alloy.
- Inlet case: Al alloy.
- Accessory section: Al or Fe alloy.
- Low Pressure Compressor, High Pressure Compressor: Ti or Ni alloy.
- Combustion chamber: Ni alloy.
- High Pressure Turbine, Turbine Blades, Low Pressure Turbine: Ni alloy.
- Turbine Exhaust Case: Ni alloy.

Superalloys  Heat resistant alloys based on Nickel, Iron-Nickel or Cobalt. For highest temperature applications, Ni-based superalloys are used.

Turbine Blade Materials

- Originally, wrought materials (forgings). Creep limited the use temperature.
- Improvements achieved by casting the blades (conventionally cast, then directionally solidified, and eventually single crystal) and using coatings.
- In the future: ceramics?

Coatings

- Two types: diffusion coatings (which are sacrificial coatings) and overlay coatings.
- Thermal Barrier Coatings are applied on turbine blades to allow higher operation temperature. They consist of a top coat (a ceramic layer, eg zirconium oxide ZrO$_2$) and a bond coat (metallic inner layer).